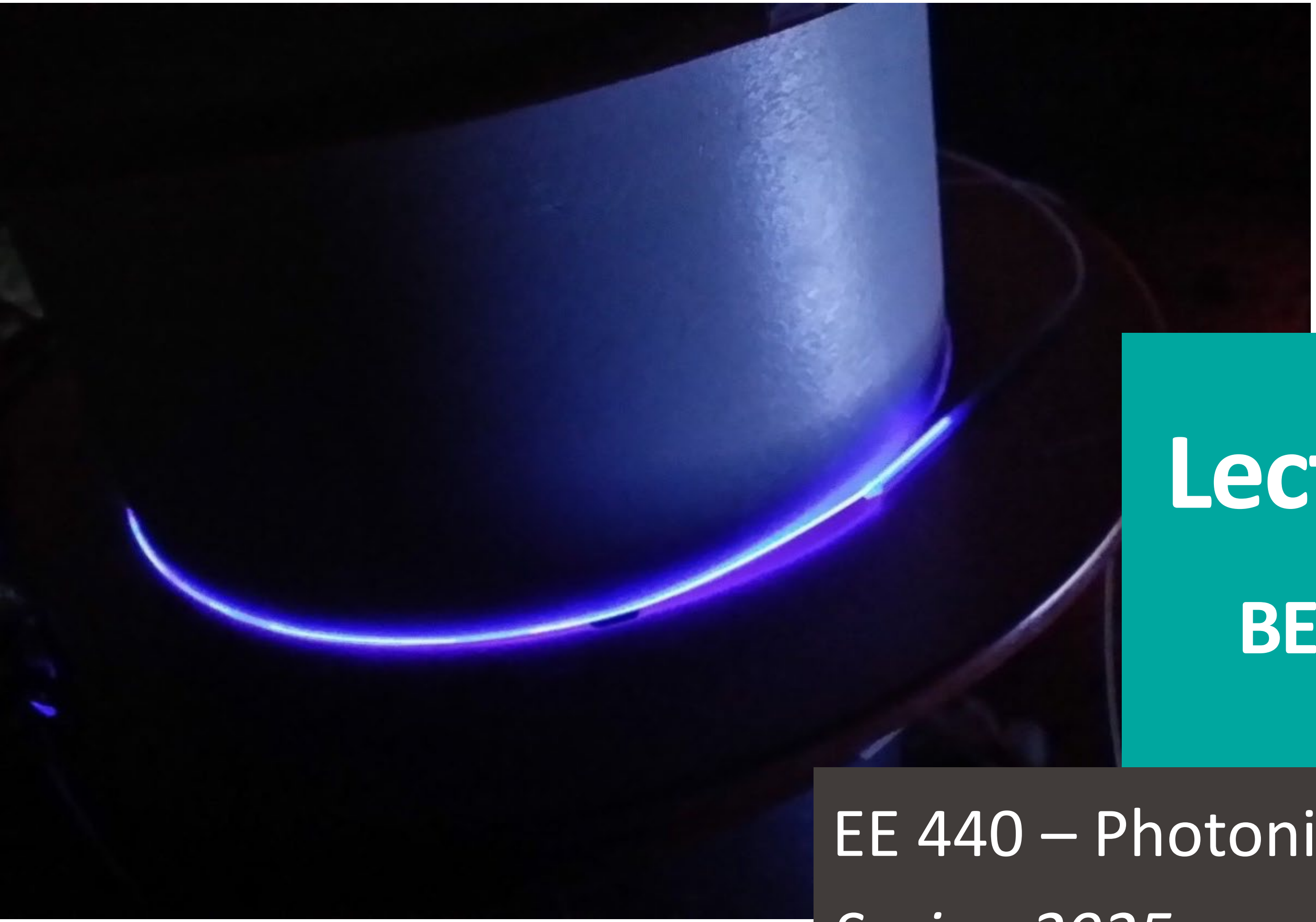

Lecture 10

BER and degradation

EE 440 – Photonic systems and technology

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Ecole
Polytechnique
Fédérale de
Lausanne

Prof. Camille Brès - camille.bres@epfl.ch

Estimation of the BER

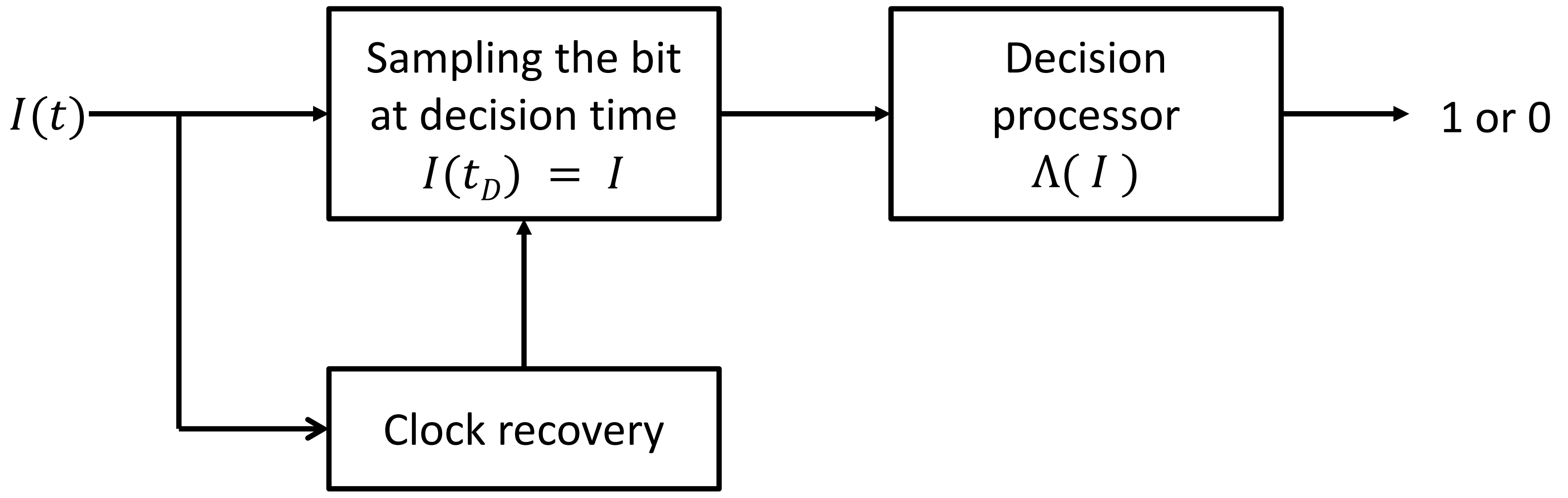
Bit error rate and receiver sensitivity

The bit error rate (BER) is the probability that a bit is incorrectly identified by the receiver.

- The error could be due to noise or signal distortion.
- A better name would be 'bit error probability'.
- A traditional requirement for optical receivers is to have BER < 10^{-9}

The receiver sensitivity is the ***minimum averaged received power*** necessary to a achieve a pre-defined target BER.

Bit error rate (BER)



Possible errors: a ‘1’ bit has been transmitted but decision processor says ‘0’

a ‘0’ bit has been transmitted but decision processor says ‘1’

Probability density function

Sampled value I fluctuates from bit to bit: $I(t) = i_{p1,p0} + i_{s1,s0}(t) + i_T(t)$

Average value $\langle I \rangle$ over a bit period

- i_{p1} if a '1' bit was sent
- i_{p0} if a '0' bit was sent

Shot noise contribution

- Described (approximated) by Gaussian statistic for p-i-n (APD)
- Zero mean, variance ${\sigma_s}^2$
- Dependent on the input signal level

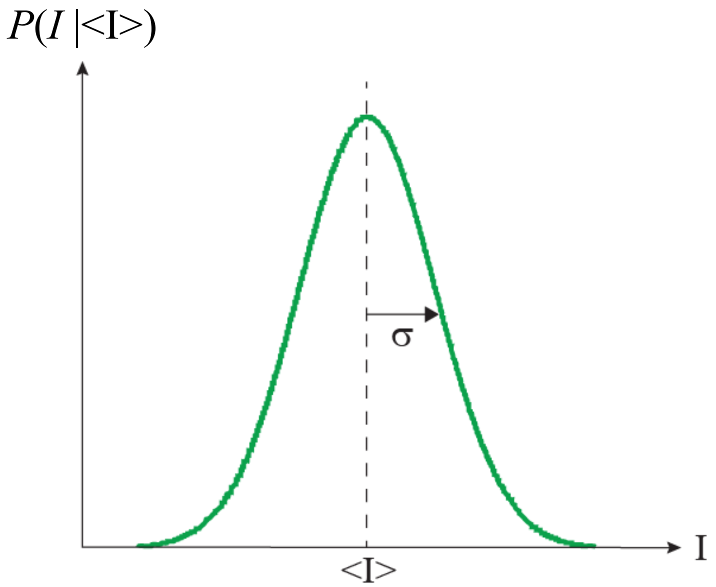
Thermal noise contribution

- Approximated by Gaussian statistic
- Zero mean, variance ${\sigma_T}^2$
- Independent from the input signal level

Sampled value and probability density function P(I |< I >)

Sampled value I

- Gaussian distribution with mean $< I >$ and width $\sigma^2 = \sigma_S^2 + \sigma_T^2$
- $\sigma_1^2 = \sigma_{s,1}^2 + \sigma_T^2$
- $\sigma_0^2 = \sigma_{s,0}^2 + \sigma_T^2$



Written as $N(\langle I \rangle, \sigma)$

$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(I - \langle I \rangle)^2}{2\sigma^2}\right]$$

BER calculation

We define the followings:

- $p(1)$ the probability to send a ‘1’, $p(0)$ the probability to send a ‘0’
- $P(0|1)$ the probability to decide 0 from a sent 1

The overall probability of error is therefore

$$\Pr(\text{error}) \equiv \text{E}\{\text{BER}\} = p(1)P(0|1) + p(0)P(1|0)$$

$$\text{BER} \approx p(1)P(0|1) + p(0)P(1|0)$$

For $p(1) = p(0) = 0.5$, such as for pulse code modulation (PCM) have:

$$\text{BER} = \frac{1}{2}[P(0|1) + P(1|0)]$$

Decision function

What are the conditional probabilities $P(0|1)$ and $P(1|0)$?

- They will depend on the probability density functions of the sampled current value I.

$$P(I|0) = P(1|i_{p0}) \quad \text{and} \quad P(I|1) = P(1|i_{p1})$$

The decision function $\Lambda(\ I\ )$ takes the sampled current value I and has to decide if it corresponds to a '1' bit or a '0' bit

- For $p(1) = p(0) = 0.5$ it is given by the likelihood ratio :

$$\Lambda(I) \equiv \frac{P(I|1)}{P(I|0)} \begin{matrix} >^{1} \\ <^{0} \end{matrix} 1$$

Single threshold test

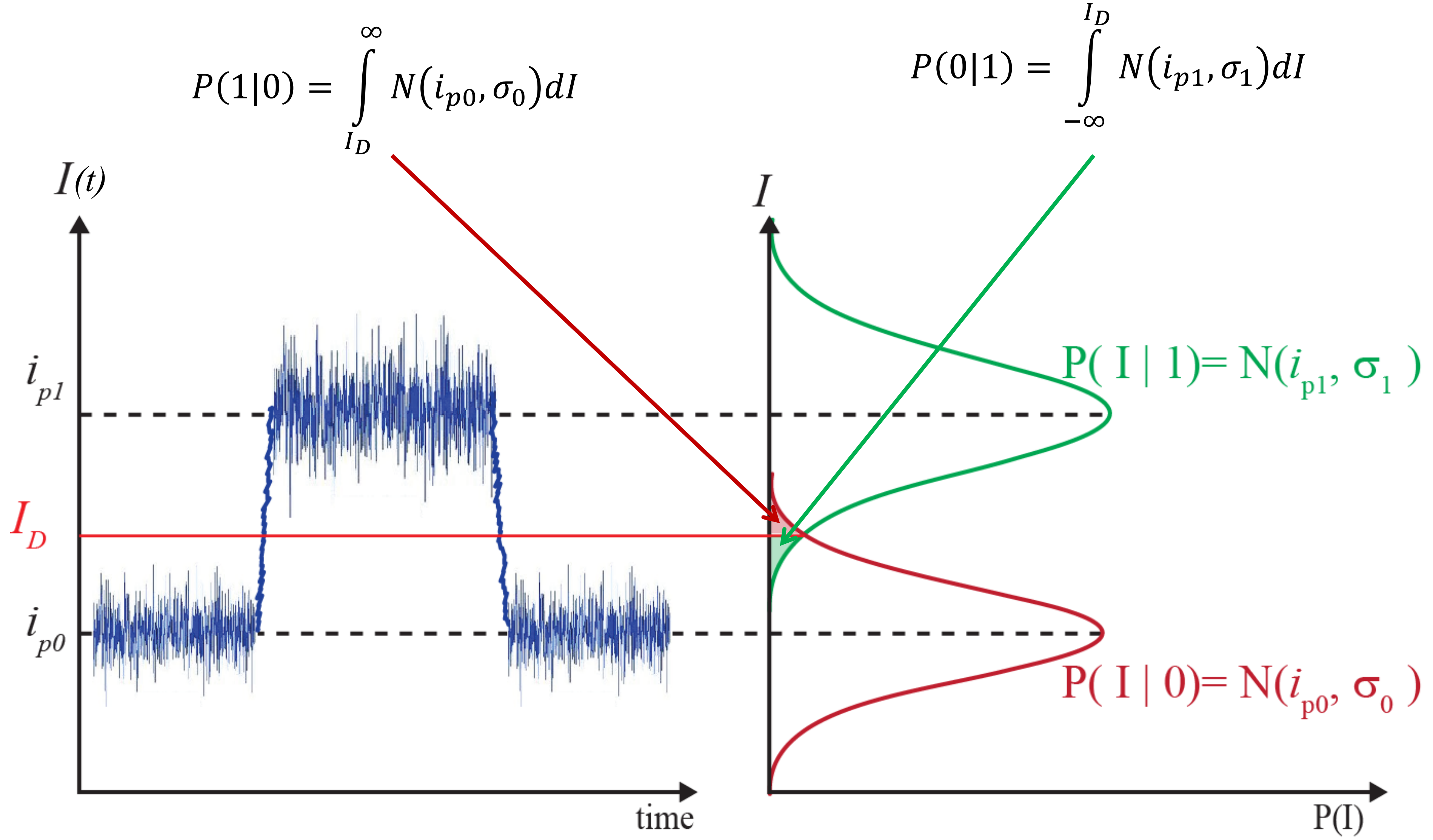
For unimodal noise distributions:

- likelihood ratio test is equivalent to comparing the sampled current value to a single current threshold value I_D , predetermined.
- The threshold value is at the intersection of the two distributions:

$$I_D \Longrightarrow P(I_D|1) = P(I_D|0)$$

Likelihood ratio $\frac{P(I|1)}{P(I|0)} \underset{0}{\overset{1}{\gtrless}} 1 \quad \Longleftrightarrow \quad \begin{array}{l} I > I_D \longrightarrow 1 \\ I < I_D \longrightarrow 0 \end{array}$ *Threshold test*

Calculating error probabilities



BER calculation

$$P(1|0) = \int_{I_D}^{\infty} N(i_{p0}, \sigma_0) dI = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} \exp\left[-\frac{(I - i_{p0})^2}{2\sigma_0^2}\right] dI = \frac{1}{2} \operatorname{erfc}\left(\frac{I_D - i_{p0}}{\sigma_0\sqrt{2}}\right)$$

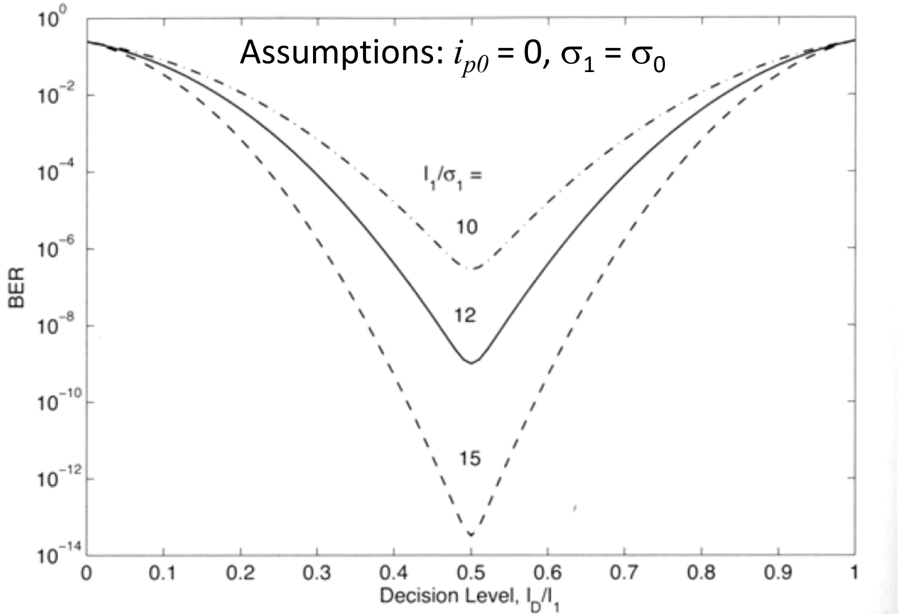
$$P(0|1) = \int_{-\infty}^{I_D} N(i_{p1}, \sigma_1) dI = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} \exp\left[-\frac{(I - i_{p1})^2}{2\sigma_1^2}\right] dI = \frac{1}{2} \operatorname{erfc}\left(\frac{i_{p1} - I_D}{\sigma_1\sqrt{2}}\right)$$

$$\text{with } \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-y^2)\, dy$$

BER calculation

$$\mathrm{BER} = \frac{1}{2}[P(0|1) + P(1|0)]$$

$$BER = \frac{1}{4}\left[\operatorname{erfc}\left(\frac{i_{p1} - I_D}{\sigma_1\sqrt{2}}\right) + \operatorname{erfc}\left(\frac{I_D - i_{p0}}{\sigma_0\sqrt{2}}\right)\right]$$



BER depends on threshold value I_D

- Also note that in general σ_1 and σ_0 are not equal

Threshold level

Minimize the BER using $\frac{d(BER)}{dI_D} = 0$

- Optimal value is at the intersection of the PDF for the 'one' and 'zero' levels
- Exact expression is given in the book

A good approximation is obtained when the probability of the two types of errors are equal:

$$P(0|1) = P(1|0) \Rightarrow \mathrm{erfc}\left(\frac{i_{p1} - I_D}{\sigma_1\sqrt{2}}\right) = \mathrm{erfc}\left(\frac{I_D - i_{p0}}{\sigma_0\sqrt{2}}\right)$$

$$\frac{i_{p1} - I_D}{\sigma_1} = \frac{I_D - i_{p0}}{\sigma_0}$$

$$I_D = \frac{\sigma_0 i_{p1} + \sigma_1 i_{p0}}{\sigma_0 + \sigma_1}$$

Threshold level : thermal noise regime

Thermal noise dominated regime ($\sigma_s^2 \ll \sigma_T^2$):

$$\sigma_0^2 = \sigma_1^2 = \sigma_T^2$$

Both distributions are Gaussian with identical variance

The threshold value is therefore:

$$I_D = \frac{i_{p1} + i_{p0}}{2}$$

- Set threshold at midpoint between means.
- When shot noise cannot be neglected, the threshold value shifts towards the ‘zero’ level.

Q factor of a transmission

Q factor and signal to noise ratio

The Q factor is defined as : $\frac{i_{p1} - I_D}{\sigma_1} = \frac{I_D - i_{p0}}{\sigma_0} \equiv Q$

Q factor can be written in terms of signal levels & noise standard deviation using the expression for the threshold current:

$$Q = \frac{i_{p1} - i_{p0}}{\sigma_1 + \sigma_0}$$

The Q factor is therefore a quantity that can be directly estimated from a graphical examination of the eye diagram as displayed on an oscilloscope

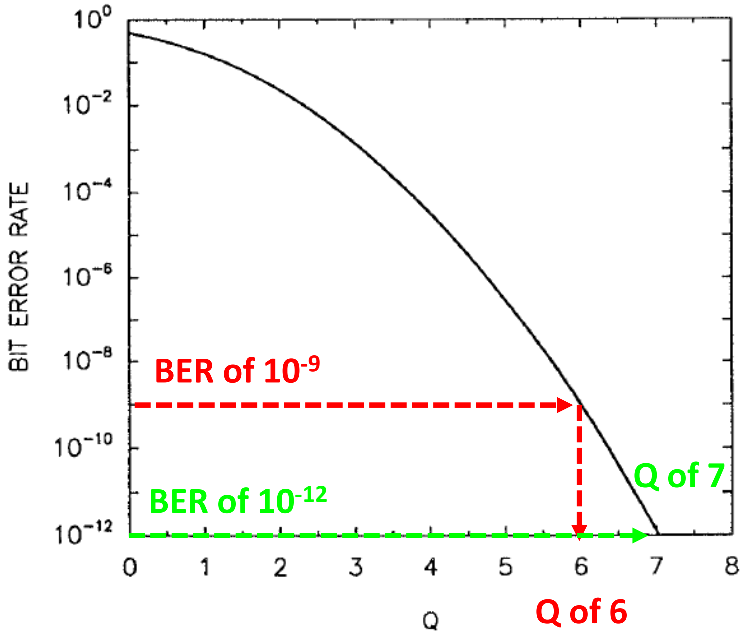
The Q factor and BER

We have defined $$\frac{i_{p1} - I_D}{\sigma_1} = \frac{I_D - i_{p0}}{\sigma_0} \equiv Q$$

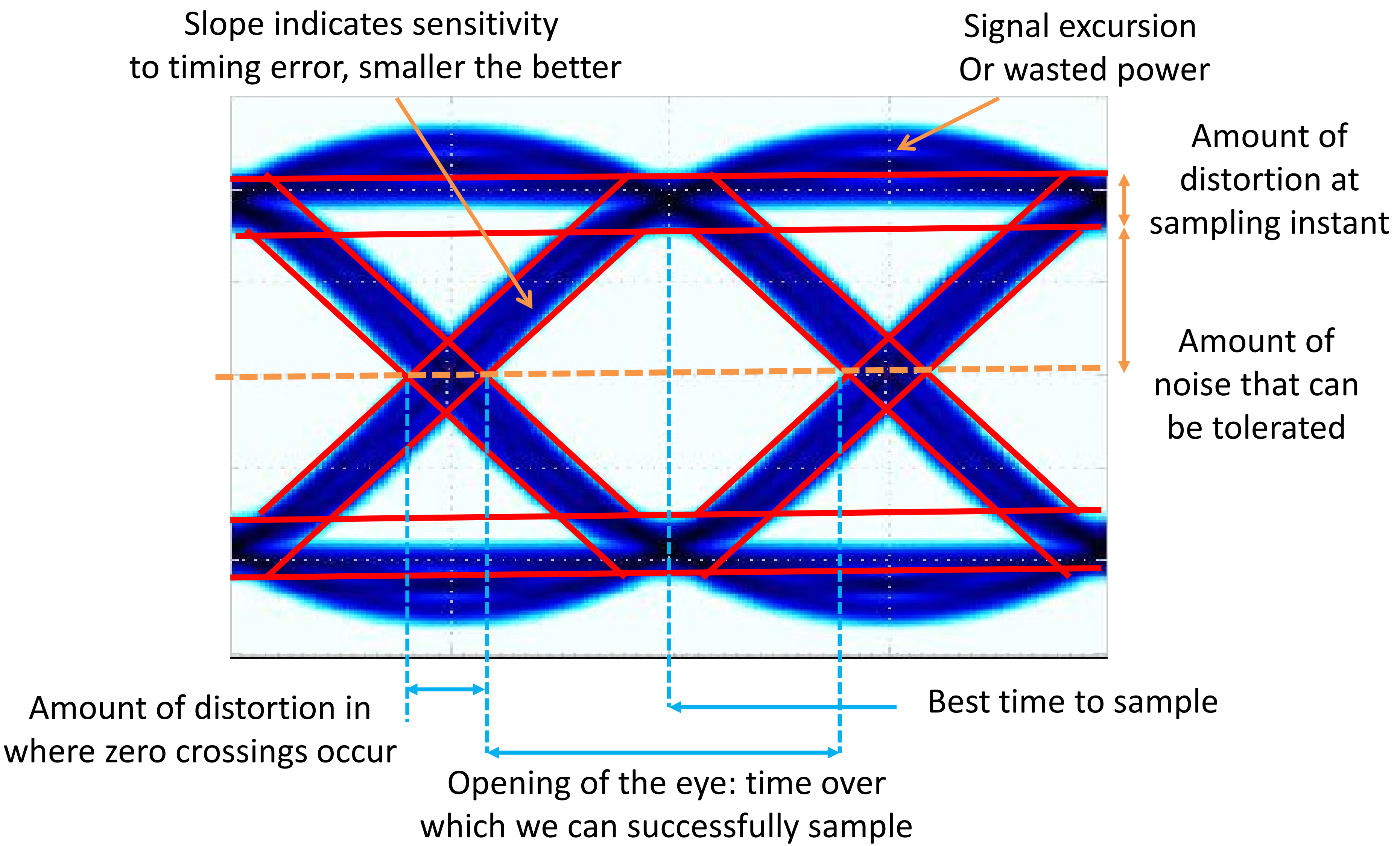
Can rewrite $$BER = \frac{1}{4}\left[\mathrm{erfc}\left(\frac{i_{p1} - I_D}{\sigma_1\sqrt{2}}\right) + \mathrm{erfc}\left(\frac{I_D - i_{p0}}{\sigma_0\sqrt{2}}\right)\right] \Rightarrow \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

For $Q > 3$:

$$BER \approx \frac{\exp\left(-\frac{Q^2}{2}\right)}{Q\sqrt{2\pi}}$$



Eye diagram interpretation



Q factor and signal to noise ratio (SNR)

A (not very accurate) BER estimate can be derived from the electrical signal-to-noise power ratio.

- Thermal noise limit (assuming $i_{p0} = 0$)

$$Q = \frac{i_{p1}}{2\sigma_T} = \frac{i_{p1}}{2\sigma_1}$$

$$SNR = \left(\frac{i_{p1}}{\sigma_1}\right)^2 \Rightarrow SNR = 4Q^2$$

$$Q = \frac{1}{2}\sqrt{SNR}$$

- Shot noise limit (assuming $i_{p0} = 0$)

$$Q = \frac{i_{p1}}{\sigma_1}$$

$$SNR = \left(\frac{i_{p1}}{\sigma_1}\right)^2 \Rightarrow SNR = Q^2$$

$$Q = \sqrt{SNR}$$

Receiver sensitivity

Receiver sensitivity $\overline{P}_{rec}$

The receiver sensitivity $\overline{P}_{rec}$ depends on:

- the desired BER (or Q factor).
- the characteristic of the receiver.
- the characteristic of the signal sent over the link.

Let's consider the following case:

- NRZ data in which the 'zero' bits contain no optical power ($P_0 = 0$)

$$\overline{P}_{rec} = \frac{P_0 + P_1}{2} \Longrightarrow P_1 = 2\overline{P}_{rec}$$

- We can neglect dark current ($i_d = 0$ A)
- Receiver uses an APD (the p-i-n case can be obtained by setting $M = F_A = 1$)

Receiver sensitivity $\overline{P}_{rec}$

Characteristics of the 'one' bits

- Average current: $i_{p1} = MRP_1 = 2MR\overline{P}_{rec}$
- Shot noise $\sigma_{s1}^2 = 2qM^2F_AR(2\overline{P}_{rec})\Delta f$
- Thermal noise $\sigma_T^2 = \dfrac{4k_BTF_n}{R_L}\Delta f$

$$\sigma_1 = \sqrt{\sigma_{s1}^2 + \sigma_T^2}$$

Characteristics of the 'zero' bits

- Average current: $i_{p0} = MRP_0 = 0$
- Shot noise : 0
- Thermal noise $\sigma_T^2 = \dfrac{4k_BTF_n}{R_L}\Delta f$

$$\sigma_0 = \sigma_T$$

Receiver sensitivity $\overline{P}_{rec}$

We know that: $Q = \dfrac{i_{p1} - i_{p0}}{\sigma_1 + \sigma_0}$

We therefore have that: $Q = \dfrac{i_{p1}}{\sigma_1 + \sigma_0} = \dfrac{2MR\overline{P}_{rec}}{\sqrt{\sigma_{s1}^2 + \sigma_T^2} + \sigma_T}$

$$\overline{P}_{rec} = \frac{Q}{R}\left(qF_A Q\Delta f + \frac{\sigma_T}{M}\right)$$

Receiver sensitivity for p-i-n

When thermal noise dominates in a p-i-n receiver:

$$\left(\overline{P}_{rec}\right)_{pin,T} \approx \frac{Q\sigma_T}{R} \propto \sqrt{\Delta f}$$

When shot noise dominates in a p-i-n receiver:

$$\left(\overline{P}_{rec}\right)_{pin,S} = \frac{q\Delta f}{R} Q^2 \propto \Delta f$$

Optimum sensitivity on APD receivers

In a receiver dominated by thermal noise, APD will increase the SNR

There is an optimum gain, given by:

$$M_{opt} = k_A^{-\frac{1}{2}} \left(\frac{\sigma_T}{Qq\Delta f} + k_A - 1 \right)^{1/2} \approx \left(\frac{\sigma_T}{k_A Qq\Delta f} \right)^{1/2}$$

The corresponding sensitivity is:

$$\left(\overline{P}_{rec} \right)_{APD} = \frac{2q\Delta f}{R} Q^2 \left(k_A M_{opt} + 1 + k_a \right)$$

Degradation and power penalty

Power penalty

Anything that *degrades* the system performance by departing from ideal conditions tends to increase the required $\overline{P}_{rec}$ for a given BER

⟹ Power penalty

Extinction ratio

- Energy carried by the '0' bits (limited modulator extinction ratio)

Intensity noise

- Light from transmitter will exhibit power fluctuations

Timing jitter

- Fluctuation of the sampling time

Extinction ratio

The extinction ratio (ER) is defined as $r_{ex} = {}^{P_0}/_{P_1}$

- P_0 (P_1) emitted power in off (on) state
- Ideally should be 0

We use the facts that:

- By definition the average received power is $\bar{P}_{rec} = \frac{P_0 + P_1}{2}$
- Q parameter is $Q = \frac{i_{p1} - i_{p0}}{\sigma_1 + \sigma_0}$

We find that the sensitivity degradation is

$$Q = \left(\frac{1 - r_{ex}}{1 + r_{ex}}\right)\frac{2R\bar{P}_{rec}}{\sigma_1 + \sigma_0}$$

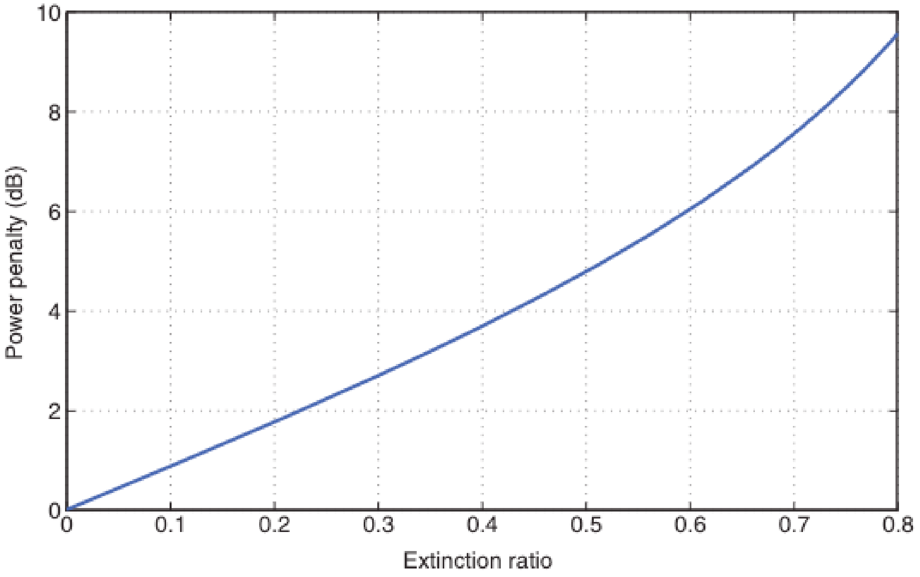
Extinction ratio and power penalty

The power penalty in dB is by definition

$$\delta_{ex} = 10\log_{10}\left[\frac{\overline{P}_{rec}(r_{ex} > 0)}{\overline{P}_{rec}(r_{ex} = 0)}\right]$$

If thermal noise dominates

$$Q = \left(\frac{1 - r_{ex}}{1 + r_{ex}}\right)\frac{R\overline{P}_{rec}}{\sigma_T} \Longrightarrow \overline{P}_{rec} = \left(\frac{1 + r_{ex}}{1 - r_{ex}}\right)\frac{\sigma_T Q}{R} \Longrightarrow 10\log_{10}\left[\frac{1 + r_{ex}}{1 - r_{ex}}\right]$$



- Shows how much the received optical power has to be increased to maintain BER
- A 1dB penalty occurs for r_{ex} = 0.12. Increased to 4.8 dB for r_{ex} = 0.5
- In practice, for laser biased below threshold r_{ex} < 0.05.
- Penalty is larger for APDs

Intensity noise

Light emitted by any transmitter exhibits power fluctuations called intensity noise

- Optical power fluctuations (with variance σ_p^2) are converted to current fluctuation (with variance σ_I^2)
- Adds to those from shot and thermal noise

$$\sigma^2 = \sigma_S^2 + \sigma_T^2 + \sigma_I^2$$

$$\sigma_I = R\left\langle\left(\Delta P_{in}^2\right)\right\rangle^{\frac{1}{2}} \equiv RP_{in}r_I$$

r_I *is the intensity noise parameter*

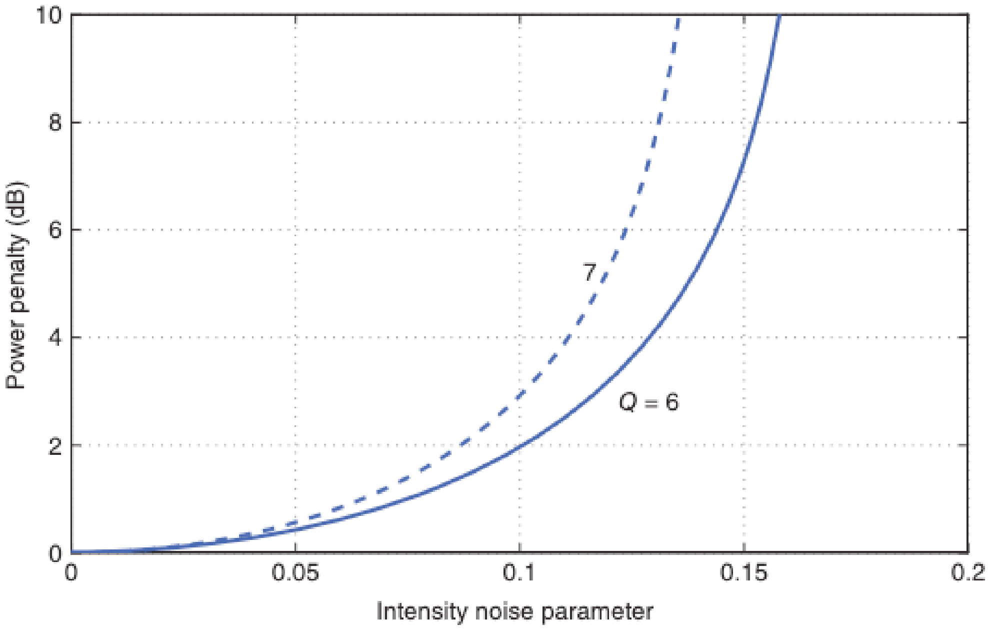
- The intensity noise parameter is related to the relative intensity noise (RIN) of a laser

$$r_i^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} RIN(\omega)d\omega$$

Power penalty due to RIN

Assuming r_{ex} = 0: P_0 = 0, and $P_1 = 2\overline{P}_{rec}$

$$\overline{P}_{rec}(r_i) = \frac{\sigma_T Q + q\Delta f Q^2}{R(1 - r_I^2 Q^2)} \implies \delta_{r_I} = 10\log_{10}\left[\frac{1}{1 - r_I^2 Q^2}\right]$$



RIN wall:

- Even infinite power will not yield desired BER if r_I is too large .

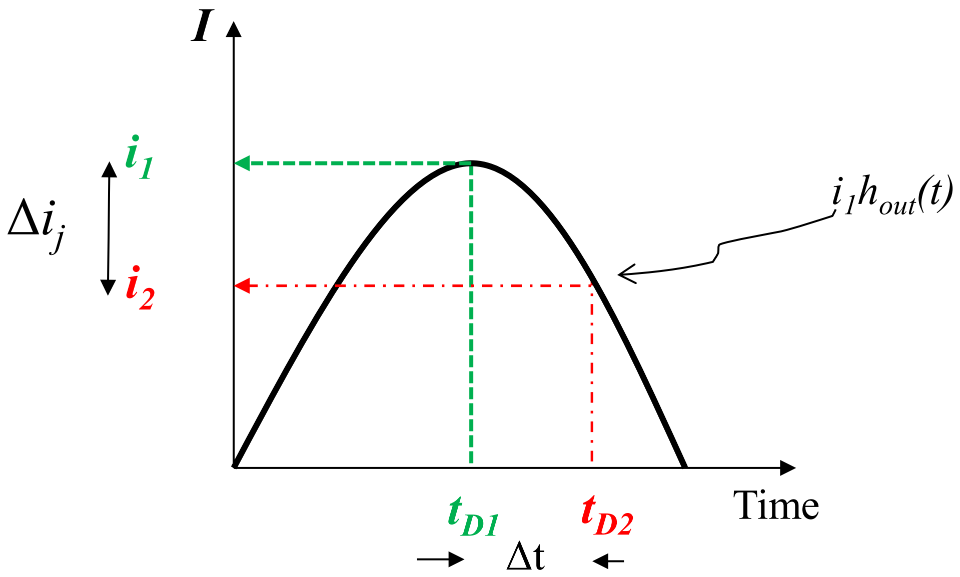
For r_I = 0.167, the BER cannot be reduced below 10^{-9}

- BER saturation is called a BER floor.

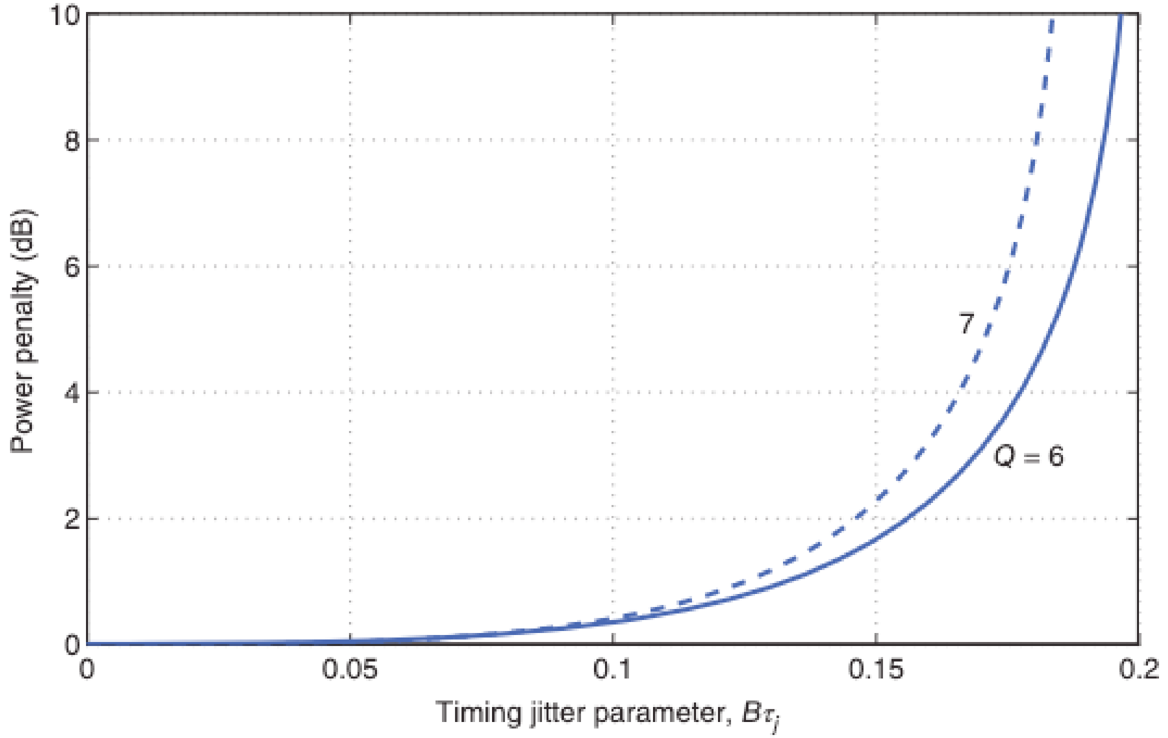
Timing jitter

Assumed that the sampled point was at the bit center

- Decision instant is determined by the clock recovery circuit
- Since input to clock recovery is noisy, **sampling time fluctuates** bit to bit by timing jitter Δt
- If bit not sampled at center, sampled value of a ’1’ bit might be reduced by an amount that depend on Δt



Timing jitter penalty



Parameter $B\tau_j$

- τ_j is RMS value of the timing jitter Δt
- $B\tau_j$ is fraction of bit period over which decision time fluctuates

Penalty is negligible for $B\tau_j < 0.1$

- Want standard deviation jitter less than 10% of bit period

Penalty becomes infinite (jitter wall) for $B\tau_j > 0.2$

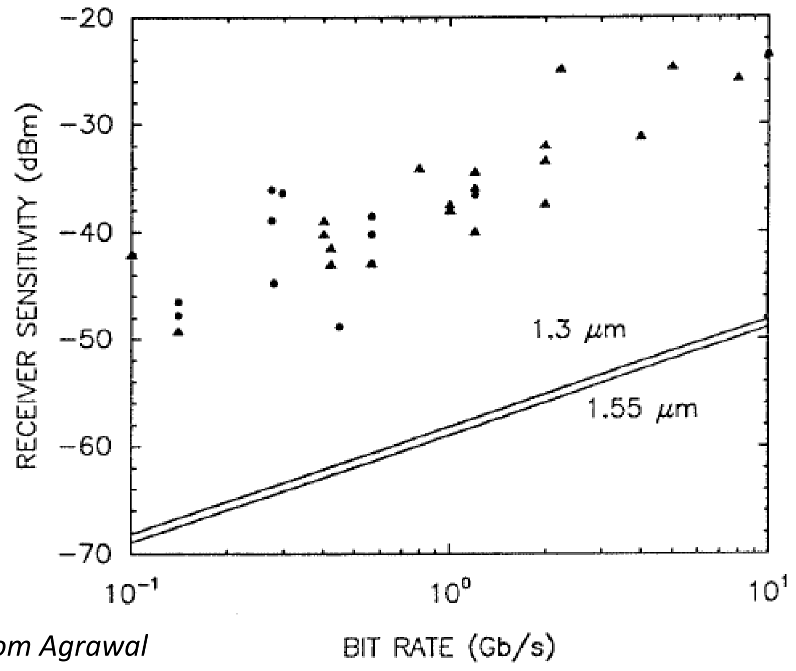
Measuring receiver performance

Simulate pseudo-random (uncorrelated) string of bits

- Use a bit error rate tester (BERT)

Sensitivity versus bit rate

- Measure BER as a function of average received power
- Sensitivity is average received power for a BER of 10^{-9}



From Agrawal

Further sources of power penalty

The previously mentioned power penalties were all due to the transmitter and the receiver.

Several more sources of power penalty appear during propagation

- Modal noise (in multi mode fibers)
- Mode partition noise (in multi mode lasers)
- Intersymbol interference (ISI) due to pulse broadening
- Frequency chirp
- Reflection feedback
- …

All of these involve dispersion.

Measuring penalty from a link

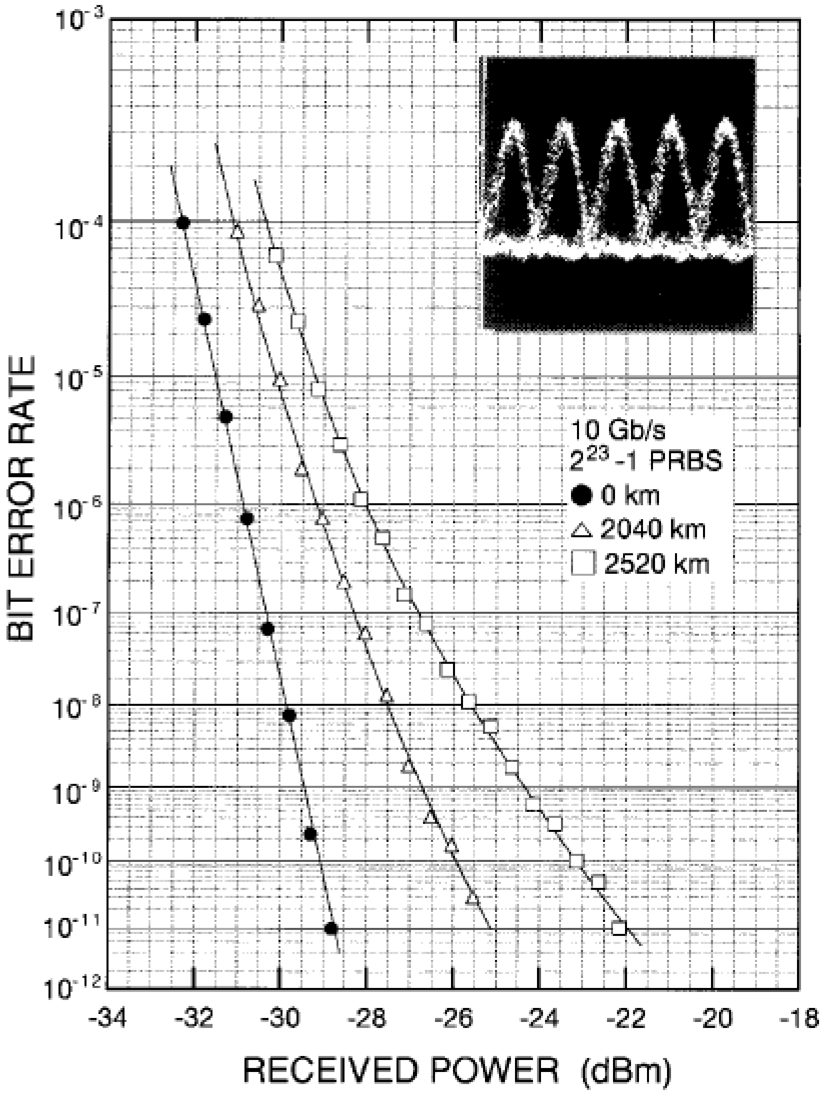
First measure back-to-back performance

- Connect transmitter directly to receiver

Then connect transmitter to the link back to the receiver

BER curves will shift due to losses, dispersion etc.

- Power penalty is estimated by the shift of the curves at a given BER



Conclusion

Optical receiver converts the optical signal back to electrical form

- Main component is photodetector
- p-i-n (or APD for low power systems) are the most common

Impairments of real receiver

- Noise: vertical closure of the eye and increased BER
  - Electronic noise: thermal noise (mostly from R_L and preamplifier), shot noise
  - Fluctuation in APD gain
  - Signal related impairments (initially noisy signal)
- Timing jitter: horizontal eye closure and increased BER
  - Imperfect clock recovery

The continuous monitoring of the eye diagram (Q factor) is common in actual systems as a measure of performance.